



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

$x_1, x_2, \dots, x_8$ , as obtained from this system of equations.

Differentiating these equations, we have respectively,

$$\left(\frac{dr_1}{d\theta_1}\right)^2, \left(\frac{dr_2}{d\theta_2}\right)^2, \dots, \left(\frac{dr_8}{d\theta_8}\right)^2 = \frac{b^2 e^4 \sin^2 \theta_1 \cos^2 \theta_1}{(1-e^2 \cos^2 \theta_1)^3}, \frac{b^2 e^4 \sin^2 \theta_2 \cos^2 \theta_2}{(1-e^2 \cos^2 \theta_2)^3}, \\ \dots, \frac{b^2 e^4 \sin^2 \theta_8 \cos^2 \theta_8}{(1-e^2 \cos^2 \theta_8)^3} \dots (2).$$

By means of the formula for the rectification of plane curves represented by polar co-ordinates, we have from (2)

$$\int_0^{r_1} ds_1 = b \int_0^{\theta_1} \frac{\sqrt{[1-e^2(2-e^2)\cos^2 \theta_1]}}{(1-e^2 \cos^2 \theta_1)^{\frac{3}{2}}} d\theta_1; \\ \int_0^{r_2} ds_2 = b \int_0^{\theta_2} \frac{\sqrt{[1-e^2(2-e^2)\cos^2 \theta_2]}}{(1-e^2 \cos^2 \theta_2)^{\frac{3}{2}}} d\theta_2; \\ \int_0^{r_3} ds_3 = b \int_0^{\theta_3} \frac{\sqrt{[1-e^2(2-e^2)\cos^2 \theta_3]}}{(1-e^2 \cos^2 \theta_3)^{\frac{3}{2}}} d\theta_3; \\ \int_0^{r_4} ds_4 = b \int_0^{\pi} \frac{\sqrt{[1-e^2(2-e^2)\cos^2 \theta_4]}}{(1-e^2 \cos^2 \theta_4)^{\frac{3}{2}}} d\theta_4; \\ \int_0^{r_5} ds_5 = b \int_0^{\theta_5} \frac{\sqrt{[1-e^2(2-e^2)\cos^2 \theta_5]}}{(1-e^2 \cos^2 \theta_5)^{\frac{3}{2}}} d\theta_5; \\ \int_0^{r_6} ds_6 = b \int_{\pi}^{\theta_6} \frac{\sqrt{[1-e^2(2-e^2)\cos^2 \theta_6]}}{(1-e^2 \cos^2 \theta_6)^{\frac{3}{2}}} d\theta_6; \\ \int_0^{r_7} ds_7 = b \int_{\pi}^{\theta_7} \frac{\sqrt{[1-e^2(2-e^2)\cos^2 \theta_7]}}{(1-e^2 \cos^2 \theta_7)^{\frac{3}{2}}} d\theta_7; \\ \int_0^{r_8} ds_8 = b \int_{\pi}^{2\pi} \frac{\sqrt{[1-e^2(2-e^2)\cos^2 \theta_8]}}{(1-e^2 \cos^2 \theta_8)^{\frac{3}{2}}} d\theta_8.$$

The evaluation of the thirty-two integrals indicated in (1) is a labor sufficient to discourage even a mathematical Hercules.

**6. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.**

Find the average length of all the diameters that can be drawn in a given ellipse.

**Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Let  $2r$  represent any diameter; then from the *central-polar* equation of

the ellipse,  $r^2 = \frac{b^2}{1-e^2 \cos^2 \theta}$ , we have  $2r = \frac{2b}{\sqrt{1-e^2 \cos^2 \theta}}$ ,

$$\frac{dr}{d\theta} = \frac{be^2 \sin \theta \cos \theta}{(1-e^2 \cos^2 \theta)^{\frac{3}{2}}}; \text{ and } \frac{ds}{d\theta} = b \sqrt{\left(\frac{1-e^2(2-e^2)\cos^2 \theta}{(1-e^2 \cos^2 \theta)^3}\right)}.$$

Since the number of diameters that can be drawn in an elliptic quadrant is proportional to the length of the elliptic arc bounding that quadrant, the required average length becomes

$$D=2b \int_0^{1\pi} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta]}}{(1-e^2\cos^2\theta)^{\frac{3}{2}}} d\theta \div \int_0^{1\pi} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta]}}{(1-e^2\cos^2\theta)^{\frac{3}{2}}} d\theta.$$

Representing  $e^2(2-e^2)$  by  $p$  and expanding,

$$\begin{aligned} D &= 2b \int_0^{1\pi} \frac{(1 - \frac{1}{2}p\cos^2\theta - \frac{1}{8}p^2\cos^4\theta - \frac{1}{16}p^3\cos^6\theta - \frac{1}{128}p^4\cos^8\theta - \text{etc.})}{(1-e^2\cos^2\theta)^{\frac{3}{2}}} d\theta \\ &\div \int_0^{1\pi} \frac{(1 - \frac{1}{2}p\cos^2\theta - \frac{1}{8}p^2\cos^4\theta - \frac{1}{16}p^3\cos^6\theta - \frac{1}{128}p^4\cos^8\theta - \text{etc.})}{(1-e^2\cos^2\theta)^{\frac{3}{2}}} d\theta \\ &= 2b \int_0^{1\pi} [1 + \frac{1}{2}e^2(2+e^2)\cos^2\theta + \frac{1}{8}e^4(4+12e^2-e^4)\cos^4\theta + \frac{1}{16}e^6(-8+52e^2 \\ &-10e^4+e^6)\cos^6\theta + \frac{1}{128}e^8(-272+800e^2-264e^4+56e^6-5e^8)\cos^8\theta + \text{etc.}] d\theta \\ &\div \int_0^{1\pi} [1 + \frac{1}{2}e^2(1+e^2)\cos^2\theta + \frac{1}{8}e^4(-1+10e^2-e^4)\cos^4\theta + \frac{1}{16}e^6(-15+39e^2 \\ &-9e^4+e^6)\cos^6\theta + \frac{1}{128}e^8(-261+564e^2-222e^4+28e^6-5e^8)\cos^8\theta + \text{etc.}] d\theta \\ &= 2b \left( \frac{1 + \frac{1}{4}e^2(2+e^2) + \frac{3}{8}e^4(4+12e^2-e^4) + \frac{5}{16}e^6(-8+52e^2-10e^4+e^6)}{1 + \frac{1}{4}e^2(1+e^2) + \frac{3}{8}e^4(-1+10e^2-e^4) + \frac{5}{16}e^6(-15+39e^2-9e^4+e^6)} \right), \end{aligned}$$

which is the required average length.

*Cor.*—Put  $e=\frac{1}{2}$ ; then substitute and reduce, we obtain

$$D = \frac{1218749}{1133057} \text{ of } 2b = 1.07563 \text{ times } 2b = 1.076 \times 2b = \frac{269}{250} \text{ (of the minor axis of}$$

the given ellipse).

This problem was also solved by Professors *SCHIEFFER* and *ZERR*. Professor *MATZ* sent in three different solutions.

## PROBLEMS.

### 13. Proposed by I. L. BEVERAGE, Monterey, Virginia

Find the mean values of the roots of the quadratic  $x^2 - ax + b = 0$ , the roots being known to be real, but  $b$  being unknown and positive.

### 14. Proposed by CHARLES E. MYERS, Canton, Ohio.

$\frac{1}{4}$  of all the mellons in a patch are not ripe, and  $\frac{1}{4}$  of all the mellons in the same patch are rotten, the remainder being good. If a man enters the patch on a dark night and takes a mellon at random, what is the probability that he will get a good one?

### 15. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Todhunter proposes: "From a point in the circumference of a circular field a projectile is thrown at random with a given velocity, which is such that the diameter of the field is equal to the greatest range of the projectile; prove the chance of its falling within the field, is  $C=2^{-1}-2\pi^{-1}(\sqrt{2}-1)$ ,  $=.236+$ ." Is this result perfectly correct as to fact?